

Nigel James.

MARKING CRITERIA AND STUDENT FEEDBACK

This section details the assessment criteria. The *extent* to which these are demonstrated by you determines your mark. The marks available for each criterion are shown. Lecturers will use the space provided to comment on the achievement of the task(s), including those areas in which you have performed well and areas that would benefit from development/improvement.

Common Assessment Criteria Applied	Marks available	Marks awarded
1. Research-informed Literature Extent of research and/or own reading, selection of credible sources, application of appropriate referencing conventions.		
Clear evidence of reading with excellent referencing.	10	10
2. Knowledge and Understanding of Subject Extent of knowledge and understanding of concepts and underlying principles associated with the discipline.		
Excellent work showing a wide knowledge. Had you included reference to instrumentation amplifiers in task 1 the mark would be even higher.	50	47
3. Analysis Analysis, evaluation and synthesis; logic, argument and judgement; analytical reflection; organisation of ideas and evidence.		
Very good analysis of the measurement system and transfer functions.	10	10
4. Practical Application and Deployment Deployment of methods, materials, tools and techniques; application of concepts; formulation of innovative and creative solutions to solve problems.		
You only lost marks in task 2 due to lack of 'by hand' calculation. Your Matlab work was good. Also in 4.1.1 the marginal 'ish' wasn't mentioned.	25	19
5. Skills for Professional Practice Attributes in professional practice: individual and collaborative working; deployment of appropriate media; presentation and organisation.		
An excellent well presented Report.	5	5

Assignment Mark Overall (Assessment marks are subject to ratification at the Exam Board. These comments and marks are to give feedback on module work and are for guidance only until they are confirmed.)	Late Submission Penalties (tick if appropriate)		91 %
	Up to one-week late		
	Over one week late		

al

HNC/HND Electrical and Electronic Engineering

Dynamics, Control and Instrumentation

M4X01405

Mr Linton Cook

Control Systems

John Nigel James

1603327

Contents

<u>Page</u>	<u>Contents</u>
1	Title Page
2	Contents
3	Task 1
10	Task 2
14	Task 3
15	Task 4
21	References

Task 1

It is required to determine the stress in a load bearing strut, as depicted in figure 1:

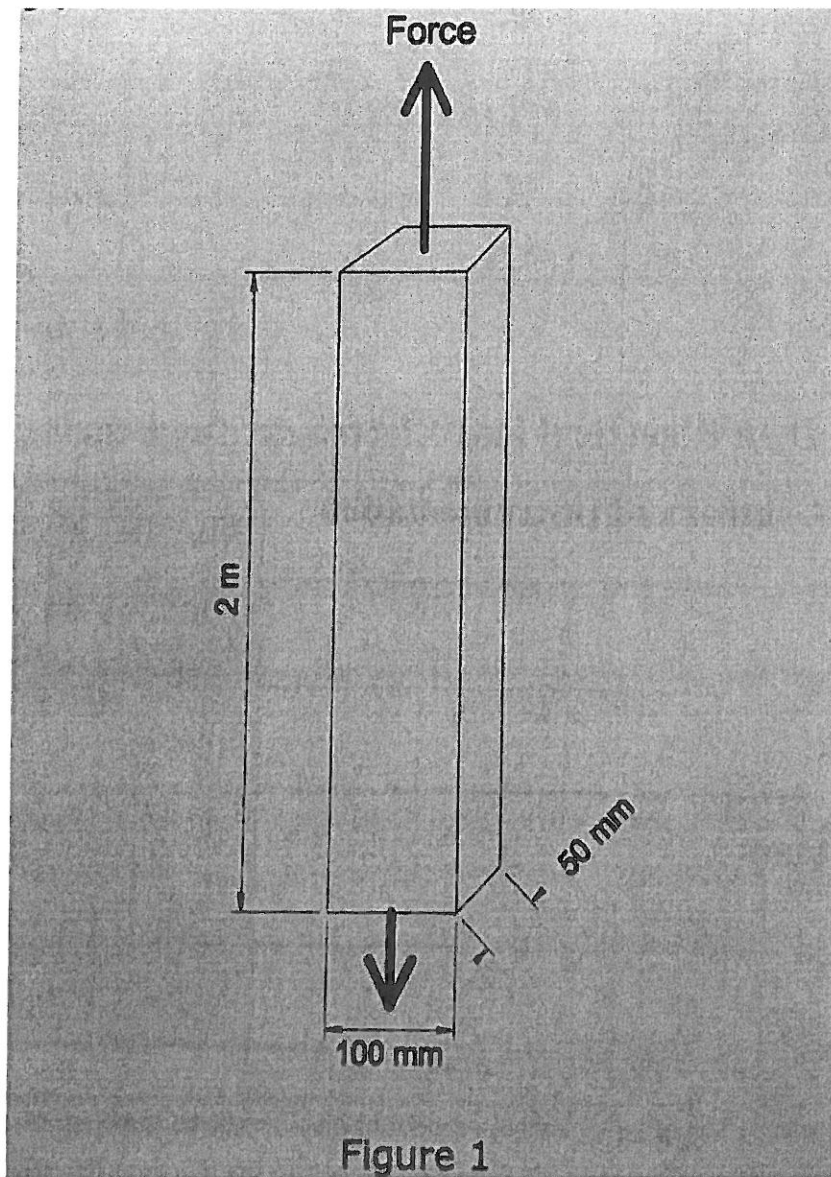


Figure 1

Task 1(a)

Propose a measurement system using strain measurements. Explain any special arrangements for maximising the quality of the measurement.

To measure the strain in the strut we need a system that will detect any physical changes in the strut and provide an electrical signal proportional to that change, which can be used to drive an indicator. This indicator can be calibrated to show these changes on a scale. Such a device is a Transducer.

Physical Effect $\rightarrow$ TRANSDUCER $\rightarrow$ Electrical Signal

In this case, the transducer would need to measure the change in the physical size of the strut as a direct result of the application of a force, to the strut. This physical change is called STRAIN (ϵ). It is a measure of the ELASTICITY of the strut as a direct result of the force, or STRESS (σ) applied to the strut.

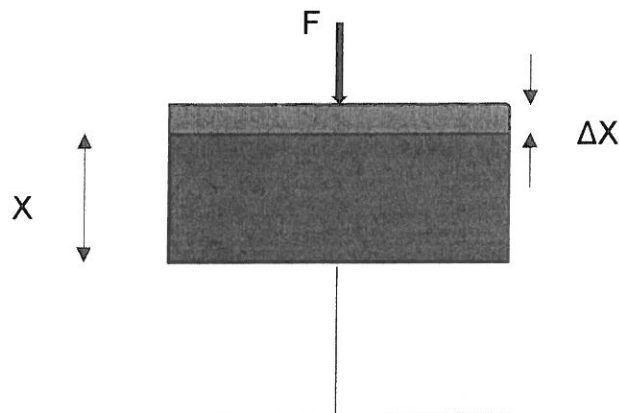


Figure 2

X = Original size of component

ΔX = Change in size of component

F = Force applied

$$STRAIN (\epsilon) = \frac{\Delta X}{X}$$

$$STRESS (\sigma) = \frac{FORCE}{AREA}$$

Such a transducer is a STRAIN GAUGE.

A strain gauge is an electrical device whose resistance varies in proportion to the amount of strain in the device. ✓

Bonded Metallic Strain Gauge

The most widely used strain gauge is of the bonded metallic type. It consists of a fine wire or metallic foil arranged in a grid pattern. This grid pattern maximises the amount of metallic wire or foil subjected to strain in a parallel direction. The grid is bonded to a fine backing paper called the carrier. This is attached directly to the strut.

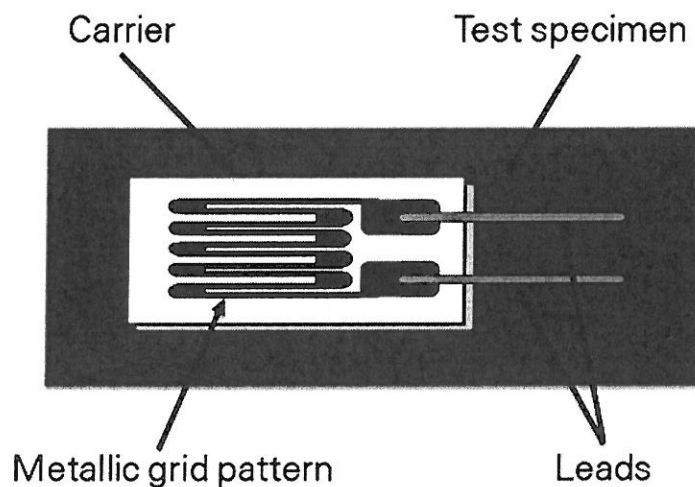


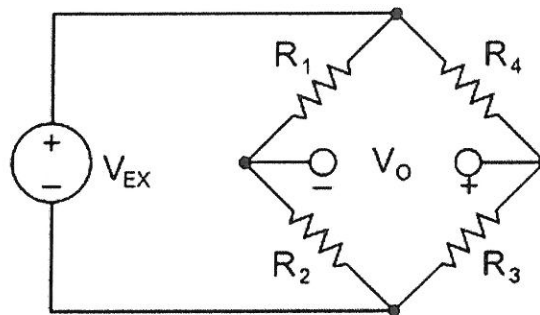
Figure 3

Therefore, any strain experienced by the strut due to the applied force will be transferred directly to the strain gauge. The resistance of the metallic wire or foil will change proportionally to the strain. This change in resistance can be used to drive the indication on a calibrated scale of a voltage or current driven display.

The change in resistance of the strain gauge element is measured in very small quantities i.e. milliohms (1×10^{-3}). Therefore, the method of converting this very small change into a usable voltage must be very accurate. The method best suited to such small measurements is to incorporate the strain gauge element into a Wheatstone Bridge. ✓

This is a network of four resistive arms with an excitation voltage (V_{EX}) applied across the bridge.

Wheatstone Bridge



This configuration is essentially two voltage divider circuits connected in parallel. R_1 and R_2 being one voltage divider and R_4 and R_3 being the other. The output of the bridge (V_0) is measured across the middle of the two dividers. The equation for this circuit is given as:

$$V_0 = \left[\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right] \times V_{EX}$$

When $R_1/R_2 = R_4/R_3$ the output of the bridge will be 0. In this condition the bridge is said to be balanced.

If R_4 in the bridge is replaced by an active strain gauge, then any changes in the resistance in the strain gauge (R_G) would unbalance the bridge and produce a voltage output (+ or -) proportional to the strain.

This can be expressed as:

$$\frac{V_0}{V_{EX}} = \frac{GF \times \epsilon}{4} \left[\frac{1}{1 + \frac{GF \times \epsilon}{2}} \right]$$

To measure the stress in the strut due the force being applied in a direction, (in this case vertically up) then the strain gauge must be fixed to the strut with the parallel elements vertically inline with the applied force. Strain in the strut in this direction would then cause the metallic elements to stretch thus altering the electrical resistance of the elements. Any strain experienced perpendicular to the applied force is called POISSONS STRAIN. This would cause the metallic elements to separate or close together. This would not alter the resistance and therefor disregard the strain experienced in this direction.

Changes in temperature of the strut and of the strain gauge could provide inaccurate measurements. To overcome this, two gauges would be applied to the strut. One in line with and one perpendicular to, the principal axis of strain but in close thermal contact of the first gauge. This is known as the quarter bridge configuration. The strain experienced by the strut would have little effect on the dummy gauge, but any temperature change would affect both equally. Thus, the ratio of their resistances would not change with temperature so would not affect the measurement. ✓ (12)

TASK 1(b)

Illustrate the performance of your system when loads of 1.5MN and 2.5MN are applied. Assuming the strut is made using steel of the dimensions shown in figure 1 with a Modulus of Elasticity of 210 Gpa.

Assuming the steel strut has a Modulus of Elasticity (E) of 210 Gpa:

- (i) For an applied load of Force (F) of 1.5MN

$$\text{Stress } (\sigma) = \frac{F}{Csa}$$

$$\text{Where } F = 1.5 \times 10^6 \text{ N}$$

$$csa = 100 \times 10^{-3} \times 50 \times 10^{-3} = 5 \times 10^{-3} \text{ m}^2$$

$$\text{Therefore Stress } (\sigma) = \frac{F}{Csa} = \frac{1.5 \times 10^6}{5 \times 10^{-3}} = 300 \text{ MPa} \quad \checkmark$$

$$\text{Strain } (\epsilon) = \frac{\Delta X}{X} = \frac{X_i - X_0}{X_0}$$

$$\text{Modulus of Elasticity (E)} = \frac{\sigma}{\epsilon}$$

$$\text{Therefore } \epsilon = \frac{\sigma}{E} \quad \text{Where } \sigma = 300 \text{ MPa} \\ E = 210 \text{ GPa}$$

$$\text{Thus } \epsilon = \frac{300 \times 10^6}{210 \times 10^9} = 1.43 \times 10^{-3}$$

$$\text{As Strain } (\epsilon) = \frac{\Delta X}{X} = \frac{X_i - X_0}{X_0} \quad \text{Then } \epsilon = \frac{\Delta X}{X}$$

$$\text{Thus } \Delta X = \epsilon \times X \quad \Delta X = 1.43 \times 10^{-3} \times 2 \text{ m} = 2.86 \times 10^{-3} = 2.86 \text{ mm} \quad \checkmark$$

(ii) For an applied load of Force (F) of 2.5MN

$$\text{Stress } (\sigma) = \frac{F}{Csa} \quad \text{Where } F = 2.5 \times 10^6 \text{ N}$$

$$Csa = 100 \times 10^{-3} \times 50 \times 10^{-3} = 5 \times 10^{-3} \text{ m}^2$$

$$\text{Therefore Stress } (\sigma) = \frac{F}{Csa} = \frac{2.5 \times 10^6}{5 \times 10^{-3}} = 500 \text{ MPa}$$

$$\text{Strain } (\epsilon) = \frac{\Delta X}{X} = \frac{X_i - X_0}{X_0}$$

$$\text{Modulus of Elasticity (E)} = \frac{\sigma}{\epsilon}$$

$$\text{Therefore } \epsilon = \frac{\sigma}{E} \quad \text{Where } \sigma = 500 \text{ MPa} \\ E = 210 \text{ GPa}$$

$$\text{Thus } \epsilon = \frac{500 \times 10^6}{210 \times 10^9} = 2.38 \times 10^{-3}$$

$$\text{As Strain } (\epsilon) = \frac{\Delta X}{X} = \frac{X_i - X_0}{X_0} \quad \text{Then } \epsilon = \frac{\Delta X}{X}$$

$$\text{Thus } \Delta X = \epsilon \times X \quad \Delta X = 2.38 \times 10^{-3} \times 2 \text{ m} = 4.76 \times 10^{-3} = 4.76 \text{ mm}$$

Therefore the application of a force of 1.5MN to the steel strut would result in a stress of 300Mpa and a strain of 0.00143. This would cause the strut to be stretched by 2.86mm.

The application of a force of 2.5MN to the steel strut would result in a stress of 500Mpa and a strain of 0.00238. This would cause the strut to be stretched by 4.76mm.

Change in resistance of a practical strain gauge of Gauge Factor (GF) of 2 and a gauge resistance of 150Ω and an input voltage (V_{EX}) of 10v.

$$GF = \frac{\Delta R}{R \times \epsilon} \text{ for } \Delta R = GF \times R \times \epsilon$$

Therefore for a load of 1.5MN where $\epsilon = 1.43 \times 10^{-3}$

$$\Delta R = GF \times R \times \epsilon \quad \Delta R = 2 \times 150 \times 1.43 \times 10^{-3} = 0.429 \Omega$$

$$\text{New resistance} = \Delta R + R = 0.429 + 150 = 150.429 \Omega$$

The output voltage (V_o) =

$$\frac{V_o}{V_{EX}} = \frac{GF \times \epsilon}{4} \left[\frac{1}{1 + \frac{GF \times \epsilon}{2}} \right]$$

$$\frac{V_o}{10} = \frac{2 \times 1.43 \times 10^{-3}}{4} \left[\frac{1}{1 + \frac{2 \times 1.43 \times 10^{-3}}{2}} \right]$$

$$\frac{V_o}{10} = 0.714 \times 10^{-3} \quad \text{thus } V_o = 7.14\text{mV}$$

For a load of 2.5MN where $\epsilon = 2.38 \times 10^{-3}$

$$\Delta R = GF \times R \times \epsilon \quad \Delta R = 2 \times 150 \times 2.38 \times 10^{-3} = 0.714 \Omega$$

$$\text{New resistance} = \Delta R + R = 0.714 + 150 = 150.714 \Omega$$

The output voltage (V_o) =

$$\frac{V_o}{V_{EX}} = \frac{GF \times \epsilon}{4} \left[\frac{1}{1 + \frac{GF \times \epsilon}{2}} \right]$$

$$\frac{V_o}{10} = \frac{2 \times 2.38 \times 10^{-3}}{4} \left[\frac{1}{1 + \frac{2 \times 2.38 \times 10^{-3}}{2}} \right]$$

$$\frac{V_o}{10} = 1.186 \times 10^{-3} \quad \text{thus } V_o = 11.86\text{mV}$$

Task 2

(a) In the context of control systems, explain the terms:

- i. Set Point
- ii. Overshoot
- iii. Plant
- iv. Control variable
- v. Controller
- vi. Control Law

a) i. The Set Point (SP) in a control system is the target value or desired outcome of a control process. Deviation from the SP can provide feed back to return the process back to the norm. ✓

ii Overshoot is the term used to describe the output of a control system which exceeds the target value or Set Point. ✓

iii Plant in control systems is the combination of the process and actuation of the system. It can be demonstrated by a transfer function showing the relationship of input to output, but without feedback. ✓

iv Control Variable is a variable that quantifies the output of the system. Control variables are used to dictate the set point. ✓

v Controller monitors the overall control process by monitoring the control variables, comparing them with the SP and using any error to feedback into the loop to eliminate any delay or overshoot and provide overall stability. ✓

vi Control Law of a control system are the rules by which the system functions, considering any feedback loops or other control variables. The control law of a system can be represented by the transfer function. ✓ (10)

b) What is meant by the term "Transfer Function" when applied to control systems? Illustrate your answer with 3 examples.

A Transfer Function of a control system is the mathematical model of the function or law of the system showing the relationship between the output to the input of a system considering feedback loops and feedforward lines. It is represented mathematically in the complex Laplace domain. ✓

A transfer function can be represented by a block diagram:



The input and output of a control system may not necessarily be of the same category. For example the input of an electric motor will be an electrical system whereas the output is mechanical. For mathematical analysis of a system the signals must be represented in a similar form. This is done by transforming the signals to LAPLACE form. ✓

A transfer Function can represented in the LAPLACE domain:

Let the input be $R(s)$ and the output be $C(s)$ then the transfer function would be:

$$G(s) = \frac{C(s)}{R(s)}$$

Examples of a Transfer Function:

(i) Mass Spring Damper:

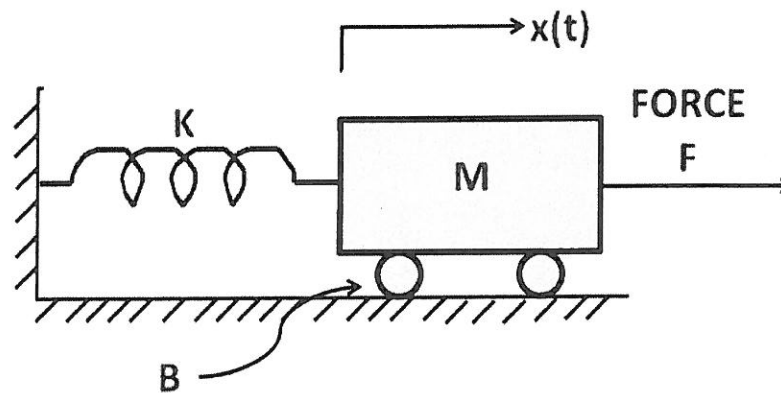


Fig-3: - A mass-spring-damper system

Where: F =	Input
X(t) =	Output (displacement from start position)
M =	Mass
K =	Restoring Force (of spring)
B =	Damping coefficient
V(t) =	Velocity

$$Gs = \frac{1}{Ms^2 + Bs + K}$$

(ii) RLC Series Network:

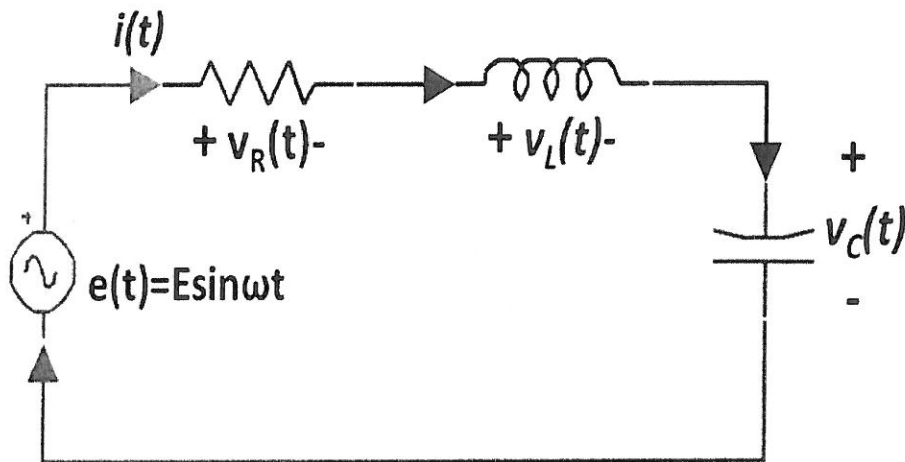


Fig 5:- RLC series circuit

$$G(s) = \frac{sC}{Ls^2 + Rcs + 1}$$

(c) Using examples, explain the key differences between “Open Loop” and “Closed Loop” control systems.

An open loop control system is a system which acts completely on the action of the input. The input and the output are unrelated and so the output has no effect on the control action. An example of an open loop control system is a washing machine. The output of the washing machine is not related to the input i.e. the cleanliness of the clothes but is dependent on a pre-selected cycle.

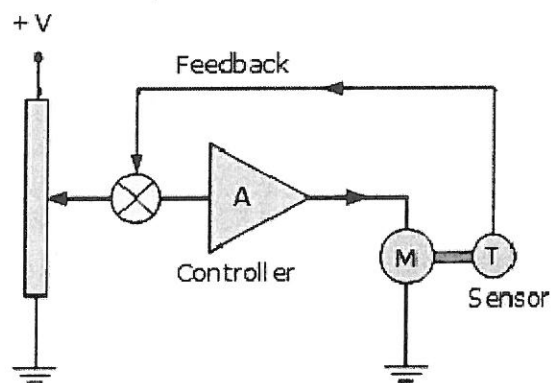
A closed loop system considers the output and compares it to the input. Any discrepancy is then applied to the control action to modify the output to the required condition. An Air Conditioning unit is an example of a closed loop system. It uses a thermostat to detect the air temperature and control the operation of the compressor to keep the room at a pre-set constant temperature.

(d) Show how closed loop systems outperform open loop systems when considering the impact of disturbances and component variation in the forward path.

In a closed loop control system any disturbance or component variation in the forward path would produce a change in the output. This change is detected and returned via the closed loop feedback line and summed with the input to produce an error signal. The error signal is then used to drive the controller to modify the output. This results in the disturbance or component variation being compensated for and the system maintained at constant output as directed by the input.

As an example of a disturbance or component variation in a closed loop control system, take the closed loop motor control as shown below. A disturbance could be an increase in the load on the motor, a component variation due to a change in temperature of the components. Either of these changes would create a difference between the actual motor speed and the speed set on the input potentiometer. This difference would produce an error signal which the controller would automatically respond to and adjust the speed of the motor. The controller works to minimise the error signal with zero error indicating actual speed which equals the set point. ✓

Closed Loop Motor Control



In an open loop system there is no feedback loop so any changes due to disturbance or component variation would remain undetected. With no error signal produced to drive the controller then the output will remain as the input plus the change due to disturbance or variation. ✓

(10)

Task 3

a) Describe sensors/transducers that may be used to measure:

- i. Linear displacement
- ii. Rotational displacement
- iii. Pressure
- iv. Presence (of a metallic item)
- v. Vibration

- I. Linear displacement can be measured by a device whose output signal is proportional to the distance travelled by an object from a reference point. Such a device is a Linear Variable Differential Transformer (LVDT). ✓
- II. Rotational (angular) displacement can be measured by a resistive displacement potentiometer. This is an electromechanical device containing an electrically conductive wiper that slides against a fixed resistive element relative to the position or angle of displacement. ✓
- III. Pressure can be measured using a pressure transducer. Such a device contains a Wheatstone bridge strain gauge. This measures the strain in the deformation of a diaphragm of a transducer caused by the pressure applied to the transducer. ✓
- IV. Presence (of a metallic item) is measured by a device called a proximity sensor. This device uses changes in the electromagnetic field caused by the proximity of metallic object to indicate its presence. ✓
- V. A Laser Doppler Vibrometer (LDV) can be used for non contact vibration measurement. The change in wavelength of the reflected laser will be proportional to the frequency of the vibration. ✓ (10)

b) Describe and compare the operation of 4-20mA and 0-10V connected transducers. What is the key advantage of 4-20mA systems?

A 4-20 mA transducer will provide an output, in mA, proportional to the change in the quantity being measured whereas the 0-10v transducer provides a proportional output in volts. A transducer with a mA range would provide an output more sensitive to smaller changes. The fact that the start range of the 4-20mA transducer is a positive quantity is an advantage as any failure in the transducer would result in the loss of the 4mA driving the indicator and highlighting the indicator. As the 0-10v transducer start range is 0v then any failure would result in no loss of indication. ✓ (5)

Task 4

- a) Explain what is meant by the term “stability” in relation to dynamic control systems.

STABILITY in a dynamic control system relates to its response to inputs to the system or other external disturbances. If a system remains in a constant state unless acted upon by an external disturbance but then returns to the constant state when the external disturbance is removed, then the system can be considered as STABLE. 

- b) Using your knowledge of stability criteria, determine the likely stability, or otherwise, for the system with the transfer functions defined below. Then, confirm your results using MATLAB. Provide copies of m-files, suitable plots and a narrative examining the plots and the conclusions that may be derived.

(i)

$$\frac{(S + 4)(S - 1)}{(S + 4)(S + 2)(S + 3)}$$

Zeros -4, 1 Poles -4, -2, -3

Numerator: $(S + 4)(S - 1) = S^2 + 3S - 4$

Denominator: $(S + 4)(S + 2)(S + 3) = S^3 + 9S^2 + 26S + 24$

So equation is
$$\frac{S^2 + 3S - 4}{S^3 + 9S^2 + 26S + 24}$$

Put into MATLAB:

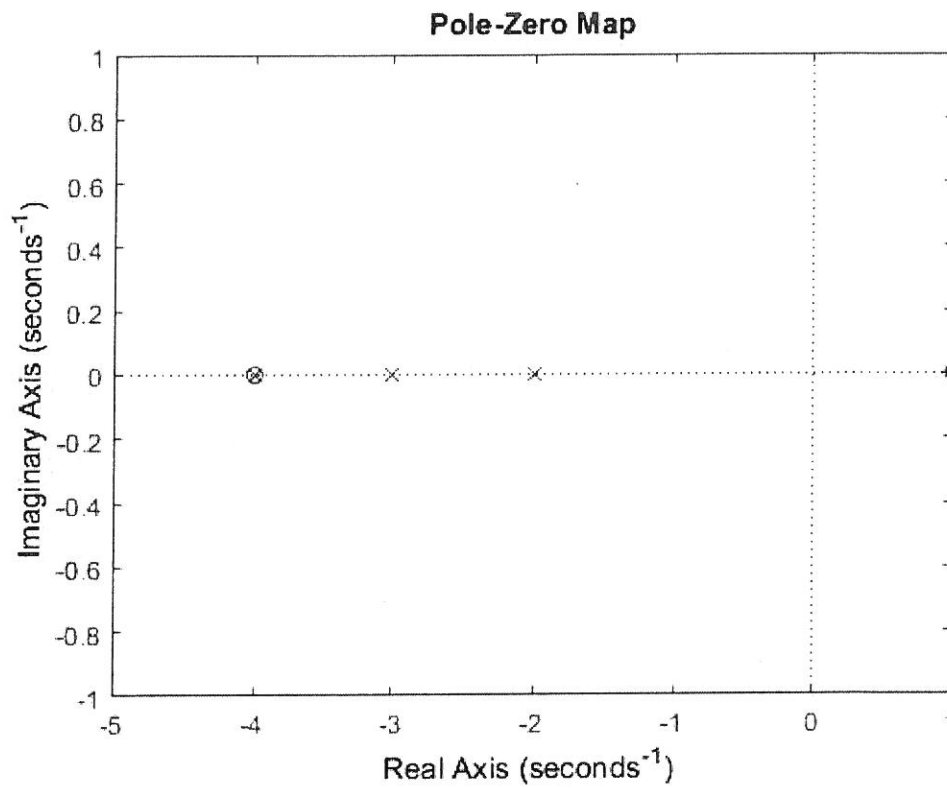
```
Q4b1=tf([1 3 -4],[1 9 26 24])
pzmap(Q4b1)
grid on
```

Q4b1 =

$$\frac{S^2 + 3s - 4}{s^3 + 9s^2 + 26s + 24}$$

Continuous-time transfer function.

Q4) (b) (i) MATLAB Pole – Zero map



As can be seen from the roots of the equation and the Pole-Zero map there are no Poles in the right-hand plane, so the system is STABLE



(ii)

$$\frac{(S-2)}{S(S+1)(S+4)}$$

Numerator: $S-2$

Denominator: $S(S+1) = (S^2+S)(S+4) = S^3+5S^2+4S$

Equation:
$$\frac{S-2}{S^3+5S^2+4S}$$

Put into MATLAB:

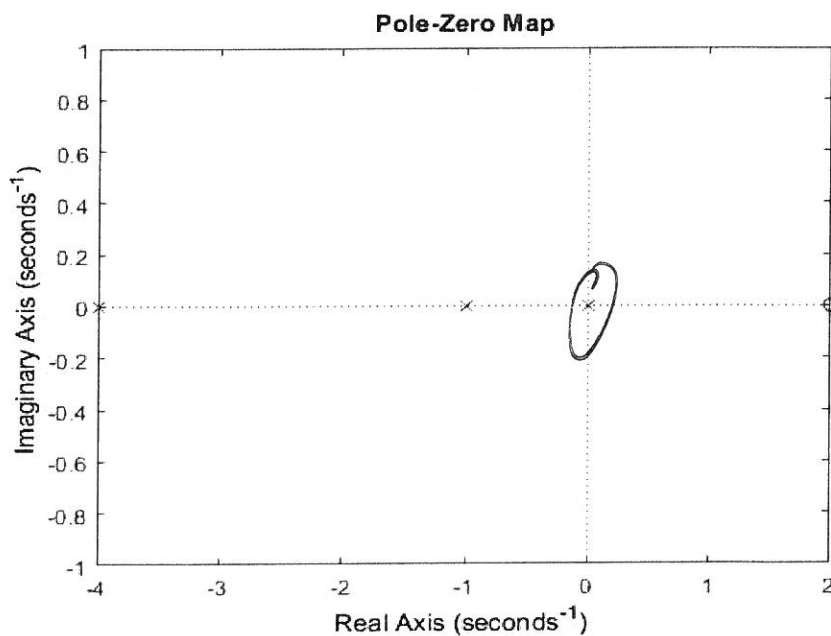
```
Q4bii = tf([0 0 1 -2],[1 5 4 0])  
pzmap (Q4bii)  
grid on
```

Q4bii =

$$\frac{s - 2}{s^3 + 5s^2 + 4s}$$

Continuous-time transfer function.

Q4) (b) (ii) MATLAB Pole – Zero map



As can be seen from the roots of the equation and the Pole-Zero map there are no Poles in the right-hand plane, so the system is STABLE

iii) $S^4 + 4S^3 - 7S^2 - 34S - 24 = 0$

It can be determined whether a system with the above polynomial equation is stable or unstable by applying the Routh-Hurwitz Criterion.

Numerator = 1

Denominator:

S^4	$1 a_4$	$-7 a_2$	$-24 a_0$
S^3	$4 a_3$	$-34 a_1$	0
S^2	b_1	b_2	0
S^1	c_1	0	0
S^0	d_1	0	0

a c e

$$\text{Det} = \frac{(b \times c) - (d \times a)}{b}$$

b d f

For b_1 : $b_1 = \frac{(b \times c) - (d \times a)}{b} = \frac{(4 \times -7) - (-34 \times 1)}{4} = \frac{6}{4} = 1.5$

By following the same method for b_2 , c_1 and d_1 matrix can be constructed as follows:

S^4	1	-7	-24
S^3	4	-34	0
S^2	1.5	-24	0
S^1	30	0	0
S^0	-24	0	0



By applying the ROUTH criterion, in that the number of sign changes in the first column of the matrix equals the number of poles in the right-side plane, then it can be seen from the matrix that there is one change in sign (highlighted in green) thus there will be one pole in the right-side plane. Therefore the system would be UNSTABLE.

Put in to MATLAB:

```
>>sysN=[1]
```

```
sysN = 1
```

```
>>sysD={1 4 -7 -34 -24}
```

```
sysD = 1 4 -7 -34 -24
```

```
>>M=tf([sysN],[sysD])
```

$$M = \frac{1}{s^4 + 4s^3 - 7s^2 - 34s - 24}$$

Calculate the roots for the equation:

```
>>Nroots = roots([sysN])
```

```
Nroots =  
0x1 empty double column vector
```

```
>>Droots = roots([sysD])
```

```
Droots =
```

```
3.0000
```

```
-4.0000
```

```
-2.0000
```

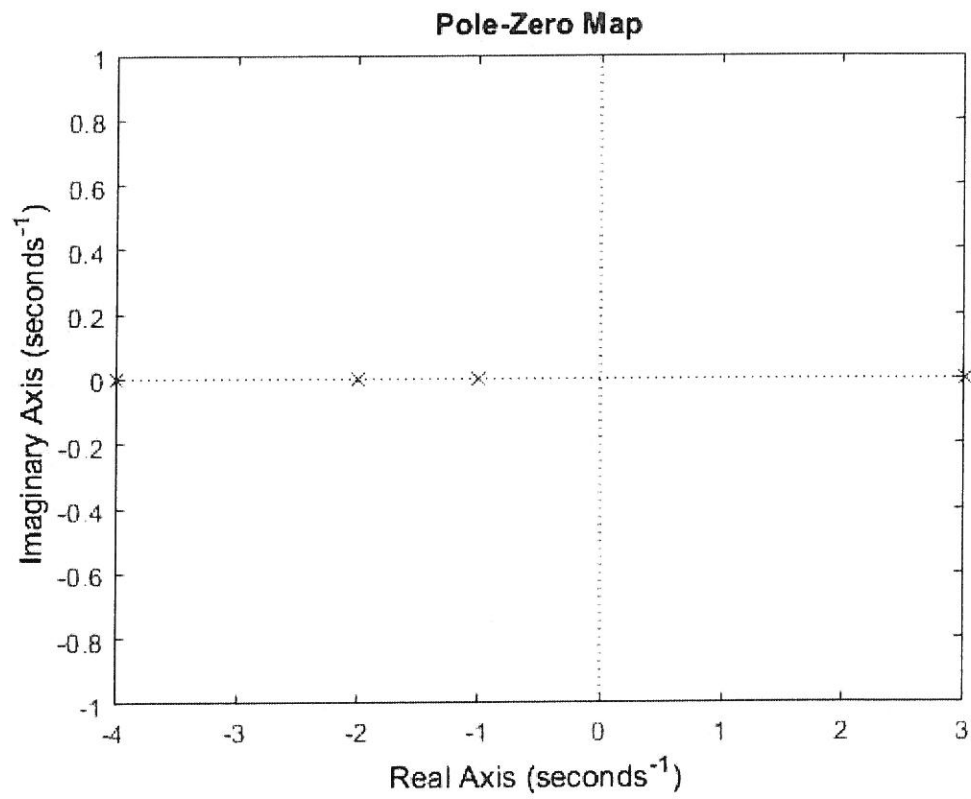
```
-1.0000
```

Create graph of POLES and ZEROS;

```
>>pzmap(M)
```

```
>>grid on
```

Q4) (b) (iii) MATLAB Pole – Zero map





References

http://moodle.gowercollegeswansea.ac.uk/pluginfile.php/166876/mod_resource/content/1/routh.pdf

<https://x-engineer.org/graduate-engineering/signals-systems/control-systems/find-transfer-function-system/>

https://www.cds.caltech.edu/~murray/courses/cds101/fa04/caltech/am04_ch6-3nov04.pdf

B. Wayne Bequette (2003). *Process Control: Modeling, Design, and Simulation*. Prentice Hall Professional. p. 5, 6

<http://www.roymech.co.uk/Related/Control/Stability.html>

https://www.tutorialspoint.com/control_systems/control_systems_stability_analysis.htm

<http://www.ni.com/white-paper/3642/en/>

<http://durofy.com/mathematical-modeling-of-control-systems-transfer-function-modeling/#prettyPhoto>

<http://web.itu.edu.tr/yildiri1/fa11ee432/exp6.pdf>

<https://www.electrical4u.com/control-system-closed-loop-open-loop-control-system/>

[file:///C:/Users/nigej/Downloads/E8 8B B1 E6 96 87 E7 89 88\(PDF\).pdf](file:///C:/Users/nigej/Downloads/E8%20B1%20E6%2096%2087%20E7%2089%2088(PDF).pdf)